MR3397482 (Review) 57R57 57M60
Sung, Chanyoung (KR-KNUE-MED)
***G*-monopole invariants on some connected sums of 4-manifolds. (English summary)**
Geom. Dedicata **178** (2015), 75–93.

Let M be a smooth 4-manifold admitting a smooth action by a compact Lie group G preserving the orientation of M and let g be a G-invariant metric on M. The action lifts to an action on a Spin^c structure $\mathfrak{s}$ on M. A lifting is fixed by a gauge map in $\mathrm{Map}(G\times M,S^1)$. The associated spinor bundles $W_\pm$ are G-invariant, so let $\Gamma(W_\pm)^G$ be the set of G-invariant sections on $W_\pm$. The superscript G means the set is composed of G-invariant elements. Thus, $\mathcal{A}^G(W_+)$ is the space of G-invariant U_1-connections on $\det(W_+)$. If an origin is fixed, this space becomes the space $\Gamma^G(\Lambda^1(M;i\mathbb{R}))$ of purely-imaginary G-invariant 1-forms taking values in $i\mathbb{R}$. Let $\mathcal{G}^G=\mathrm{Map}(M,S^1)^G$ be the group of G-invariant gauge transformations. For each G-invariant perturbation $\epsilon\in\Gamma^G(\Lambda^2_+(M;i\mathbb{R}))$, the perturbed Seiberg-Witten equation is the smooth map

$$H\colon \mathcal{A}^G(W_+)\times\Gamma^G(W_+\times\Gamma(\Lambda^2_+(M;i\mathbb{R})))\to\Gamma^G(W_-)\times\Gamma^G(\Lambda^2_-(M;i\mathbb{R}))$$

defined by

$$H(A,\Phi,\epsilon)=\left(D_A\Phi,F_A^+-\Phi\otimes\Phi+\frac{|\Phi|^2}{2}\mathrm{Id}+\epsilon\right).$$

Given $l>0$, the domain and the range are endowed with L^2_{l+1} and L^2_l Sobolev norms, respectively. The G-monopole moduli space χ_ϵ is then defined as $\chi_\epsilon=H_\epsilon^{-1}(0)/\mathcal{G}^G$, where $H_\epsilon(\cdot,\cdot)=H(\cdot,\cdot,\epsilon)$. It is proved that if G is finite and ϵ is a generic perturbation, then χ_ϵ is a smooth compact manifold in the ordinary Seiberg-Witten moduli space $\mathfrak{M}_\epsilon$. Indeed, it is remarked that χ_ϵ may not be compact for G not finite. In this way, the article computes the G-Seiberg-Witten invariant on some G-manifolds. Namely, it is proved that the connected sum of k copies of a 4-manifold whose mod 2 Seiberg-Witten invariant has non-zero $\mathbb{Z}_k$-monopole invariant mod 2, where the $\mathbb{Z}_k$-action is given by the cyclic permutations of the k summands. The main theorem is the following;

Main Theorem: Let M and N be smooth closed oriented connected 4-manifolds satisfying $b_2^+(M)>1$ and $b_2^+(N)=0$, and, for any $k\geq 2$, let $\widetilde{M}_k$ be the connected sum $M\#\cdots\#M\#N$ where there are k summands of M. Suppose that a finite group G with $|G|=k$ acts effectively on N in a smooth orientation preserving way such that it is free or has at least one fixed point, and that N admits a Riemannian metric of positive scalar curvature invariant under the G-action and a G-equivariant Spin^c structure $\mathfrak{s}_N$ with $c_1^2(\mathfrak{s}_N)=-b_2(N)$. Define a G-action on $\widetilde{M}_k$ induced from that of N permuting k summands of M glued along a free orbit in N, and let $\widetilde{\mathfrak{s}}$ be the Spin^c structure on $\widetilde{M}_k$ obtained by gluing $\mathfrak{s}_N$ and a Spin^c structure $\mathfrak{s}$ of M.

Then for any G-action on $\widetilde{\mathfrak{s}}$ covering the above G-action on $\widetilde{M}_k$, $SW^G_{\widetilde{M}_k,\widetilde{\mathfrak{s}}}$ mod 2 is nontrivial if $SW_{M,\mathfrak{s}}$ mod 2 is nontrivial.

See also [S. A. Bauer and M. Furuta, Invent. Math. **155** (2004), no. 1, 1–19; MR2025298; S. A. Bauer, Invent. Math. **155** (2004), no. 1, 21–40; MR2025299; Y. S. Cho, Acta Math. Hungar. **84** (1999), no. 1-2, 97–114; MR1696538; S. J. Baldridge, Commun. Contemp. Math. **3** (2001), no. 3, 341–353; MR1849644]. *Celso M. Doria*

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MR3373041 (Review) 57M27 57M50
László, Tamás (H-CEU); **Némethi, András** (H-AOS)
Reduction theorem for lattice cohomology. (English summary)
Int. Math. Res. Not. IMRN **2015,** *no. 11,* 2938–2985.

The authors start by considering a connected negative definite plumbing graph G. This kind of graph can be realized as the resolution graph of some normal surface singularity $(X,0)$, and the link M of $(X,0)$ can be considered as the plumbed 3-manifold associated with G. In the article, it also assumed that M is a rational homology sphere, or, equivalently, G is a tree and all the genus decorations are zero. For more details consult [A. Némethi, Geom. Topol. **9** (2005), 991–1042; MR2140997; in *Geometry and topology of manifolds*, 219–234, Fields Inst. Commun., 47, Amer. Math. Soc., Providence, RI, 2005; MR2189934; in *Singularities in geometry and topology*, 394–463, World Sci. Publ., Hackensack, NJ, 2007; MR2311495; Publ. Res. Inst. Math. Sci. **44** (2008), no. 2, 507–543; MR2426357].

The second author [op. cit.; MR2140997; op. cit.; MR2426357] associated with such M, with a fixed $spin^c$-structure $\mathfrak{s}$, a graded $\mathbb{Z}[U]$-module $H^*(M,s)$ called the lattice cohomology of M. The lattice cohomology is purely combinatorial. One of the main conjectures concerning the topic claims that $H^*(M,s)$ contains all the information about the Heegaard-Floer homology of M. The second author proved that the normalized Euler characteristic of the lattice cohomology coincides with the normalized Seiberg-Witten invariant of the link M, thus providing a new combinatorial formula for the Seiberg-Witten invariants.

The explicit computation of the lattice cohomology based on its very definition is a very hard task. A priori, it is based on the computation of the weights of all lattice points (of a certain $\mathbb{Z}^s$) and on the description of those regions where the weights are less than n for any integer n. The rank of the lattice which appears in the construction is very large; indeed, it is the number of vertices of the corresponding plumbing/resolution graph G of M. (The weight is provided by a Riemann-Roch formula.)

In order to decrease the computational complexity and also to establish the conceptual properties of the lattice cohomology one tries to decrease the rank of the lattice and simplify the graded cohomological complexes in such a way that the new presentation contains essentially no superfluous data, focusing exactly on the geometry of the 3-manifold. That is the strategy of the article to achieve the main result, called the *Reduction Theorem*. Indeed, the authors show how to reduce the rank of the lattice to the number ν of bad vertices of the plumbing. This number measures how far the graph stays from a rational graph.

The role of the Reduction Theorem is explained by the authors by the following parallelisms:

(i) Homotopy version: find a universal procedure which provides for any CW complex X a (minimal) sub-complex K such that $K \subset X$ is a homotopy equivalence.

(ii) Cohomological version: fix a cohomology theory H^*, and then find a universal procedure which provides to any X as above a (minimal) sub-complex $i\colon K \hookrightarrow X$ such that $i^*\colon H^*(X) \to H^*(K)$ is an isomorphism.

This strategy should be good enough to detect all the intrinsic properties of X.

The authors consider the lattice cohomology H^* which associates to any lattice (or a part of it, e.g., a quadrant or rectangle) and weight function (L, w) the module $H^*(L, w)$. The pair (L, w) will be associated with a plumbed 3-manifold M (constructed from the graph whose intersection lattice is L) and with a fixed *spin*^{c}-structure of M. The Reduction Theorem finds a (minimal and functorial) weighted first-quadrant in a certain sublattice $(\overline{L}, \overline{w})$ with the same cohomology, where $\overline{L}$ is the lattice generated by the bad vertices. Let $\overline{L}_{\geq 0}$ be the first quadrant of $\overline{L}$. For any fixed *spin*^{c}-structure $\mathfrak{s}$, and for any lattice point $i \in \overline{L}_{\geq 0}$, they find a very special universal point $x(i)$ in L and set $\overline{w}(i) = w(x(i))$. Then the lattice cohomology of the pair (M, s), $H^*(L, w)$ can be recovered by the isomorphism

$$H^*(L, w) = H^*(\overline{L}_{\geq 0}, \overline{w}) \quad \text{(Reduction Theorem).}$$

Celso M. Doria

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR3298669 (Review) 57N13 57R58
Horvat, Eva (SV-LJUBFE)
Double plumbings of disk bundles over spheres. (English summary)
Comm. Anal. Geom. **23** (2015), *no. 2,* 225–272.

The article's main result is the Heegaard-Floer homology computation of a double plumbing of two disk bundles over spheres. So, for a closed smooth 4-manifold X, the author investigates when a pair of classes in $H_2(X)$ may be represented by a configuration of surfaces in X whose regular neighborhood is a double plumbing of disk bundles over spheres. Indeed, she calls attention to the more broad question of finding the simplest configuration of surfaces in X representing a finite set of classes $C \subset H_2(X)$. By simple the author means of low genus and that the surfaces should have a low number of geometric intersections. She points out that, when considering a configuration of surfaces, the sum of their genera is closely related to the number of their geometric intersections. As shown by P. M. Gilmer [Trans. Amer. Math. Soc. **264** (1981), no. 2, 353–380; MR0603768], the minimal number of such intersections can be estimated by using the Casson-Gordon invariant. Let $N_{m,n}$ be the double plumbing of two disk bundles with Euler classes m and n over spheres, which represents the simplest case of a configuration of two surfaces with algebraic and geometric intersection 2.

Let $Y_{m,n} = \partial N_{m,n}$. Given the integers i, j consider $\mathfrak{t}_{i,j}$ as the unique $Spin^c$ structure on $N_{m,n}$ such that

$$\langle c_1(\mathfrak{t}_{i,j}), s_1\rangle + m = 2i, \quad \langle c_1(\mathfrak{t}_{i,j}), s_2\rangle + n = 2j,$$

where $s_1, s_2 \in H_2(N_{m,n})$ represent the homology classes of the base spheres in the plumbing. Let $\mathfrak{s}_{ij} = \mathfrak{t}_{ij}\,|_{Y_{m,n}}$, $\mathbb{F} = \mathbb{Z}_2$ and $\mathcal{T}^+ = \mathbb{F}[U, U^{-1}]/U\mathbb{F}[U]$ be a quotient module.

Main Theorem. Let $Y = Y_{m,n}$ be the boundary of a double plumbing of two disk bundles over spheres with Euler numbers m and n, where $m, n \geq 4$. The Heegaard-Floer homology $HF^+(Y, \mathfrak{s})$ with $\mathbb{F}$ coefficients is given by

$$HF^+(Y, \mathfrak{s}_{m-1,1}) = \mathcal{T}^+_{(d(m-1,1))} \oplus \mathcal{T}^+_{(d(m-1,1)-1)} \oplus \mathbb{F}_{(d(m-1,1)-1)},$$

$$HF^+(Y, \mathfrak{s}_{i,j}) = \mathcal{T}^+_{(d(i,j))} \oplus \mathcal{T}^+_{(d(i,j)-1)},$$

$$HF^+(Y, \mathfrak{s}_{0,j}) = \mathcal{T}^+_{(d_1(m,k+1))} \oplus \mathcal{T}^+_{(d_1(m,k+1)-1)},$$

$$HF^+(Y, \mathfrak{s}_{l,0}) = \mathcal{T}^+_{(d_1(n,l+1))} \oplus \mathcal{T}^+_{(d_1(n,l+1)-1)}$$

for $1 \leq i \leq m-1,\ 1 \leq j \leq n-1,\ 0 \leq k \leq n-2,\ 0 \leq l \leq m-2$ and $(i,j) \notin \{(m-1,1),(1,n-1)\}$, where the subscripts denote the absolute gradings of the bottom elements and

$$d(i,j) = \frac{m^2n + mn^2 - 4mn(i+j+1) + 4n(i^2+2i) + 4m(j^2+2j) - 16ij}{4(mn-4)},$$

$$d_1(t,i) = \frac{m^2n + mn^2 - 4mni + 4ti^2 - 4t}{4(mn-4)}.$$

The action of the exterior algebra $\Lambda^*(H_1(Y,\mathbb{Z})/\mathrm{Tors})$ on $HF^+(Y,\mathfrak{s})$ maps the first copy of $\mathcal{T}^+$ isomorphically to the second copy in each torsion $Spin^c$ structure $\mathfrak{s}$, dropping the absolute grading of the generator by one. The main theorem is applied to determine whether $N_{m,n}$ occurs inside of some 4-manifold X with $H_2^+(X) = 2$. Whenever possible, the complement $W = X \backslash Int(N_{m,n})$ is a negative semi-definite 4-manifold. In [Adv. Math. **173** (2003), no. 2, 179–261; MR1957829] P. S. Ozsváth and Z. Szabó gave an obstruction depending on the correction terms of $Y_{m,n} = \partial W$. In this way, the following theorems are obtained.

Theorem A. (a) Any two spheres representing the homology classes $(2,2)$ and $(2,-1)$ in $H_2(\mathbb{C}\mathrm{P}^2 \# \mathbb{C}\mathrm{P}^2)$ intersect with at least 4 geometric intersections, and there exist representatives with exactly 4 intersections.

(b) Let $t \in \mathbb{N} \setminus \{1\}$ and let a be an odd positive integer. Any two spheres representing classes $(a,2,0,0), (1,0,t,1) \in H_2(S^2 \times S^2 \# S^2 \times S^2)$ intersect with at least 4 geometric intersections for all $a \geq 5$.

Theorem B. Let k be a positive integer. Any two spheres representing classes $(2k+1,2,0,0), (-k,1,2k,1) \in H_2(S^2 \times S^2 \# S^2 \times S^2)$ intersect with at least 3 geometric intersections for all $k > 1$.

Celso M. Doria

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR3207377 (Review) 55R80 57R45 58E05
Baryshnikov, Yuliy [Baryshnikov, Yuliy M.] (1-IL); **Bubenik, Peter** (1-CVLS); **Kahle, Matthew** (1-IASP-SM)

Min-type Morse theory for configuration spaces of hard spheres. (English summary)

Int. Math. Res. Not. IMRN **2014,** *no. 9,* 2577–2592.

The article studies the configuration of spheres in a bounded region in $\mathcal{B} \subset \mathbb{R}^d$ by applying an extension of Morse Theory ideas to the case where the function is a special continuous function. Fix n, and define $\mathrm{Conf}(n)$ to be the set of ordered n-tuples of distinct points in $\mathcal{B}$:

$$\mathrm{Conf}(n) = \{\vec{x} = (x_1, \ldots, x_n) \mid x_i \in \mathcal{B},\ x_i \neq x_j \text{ for all } i \neq j\}.$$

For $r \geq 0$, define $\mathrm{Conf}(n, r)$ to be the configuration space of n nonoverlapping balls of radius r in $\mathcal{B}$. For r sufficiently small, $\mathrm{Conf}(n, r)$ is homotopy equivalent by a retraction to $\mathrm{Conf}(n)$. For r sufficiently large, $\mathrm{Conf}(n, r) = \varnothing$. The problem of finding the smallest such r is equivalent to solving the packing problem. Indeed, the authors determine the threshold radius below which the configuration space $\mathrm{Conf}(n, r)$ is homotopy equivalent to $\mathrm{Conf}(n)$.

Their method relies on the *tautological* function $\tau\colon \mathrm{Conf}(n) \to \mathbb{R}$ defined by

$$\tau(\vec{x}) = \min\left(\frac{1}{2} \min_{i \neq j} d(x_i, x_j), \min_i \min_{p \in \partial\mathcal{B}} d(x_i, p)\right),$$

where $\partial\mathcal{B}$ is the boundary of $\mathcal{B}$. So, the space $\mathrm{Conf}(n, r)$ is defined in terms of the tautological function as

$$\mathrm{Conf}(n,r) = \tau^{-1}[r,\infty).$$

It is remarked that the definition above suggests using a Morse type of theory associated to τ in order to study the topology of $\mathrm{Conf}(n,r)$ and how the topology changes as r varies. A technical problem arises from the fact that τ may not be smooth. This is overcome by developing a Min-Type Morse Theory. Let M be a manifold and $f\colon M \to \mathbb{R}$ a function, and let $M^c = f^{-1}[c,\infty)$. A function $h\colon (s,t) \to \mathbb{R}$ is increasing with speed at least $v > 0$ on the interval (s,t) if

$$h(t') - h(s') \geq v(t'-s'), \text{ for any } s' < t'.$$

For a smooth vector field V on M, the authors denote the time t shift along the trajectories of V as S_t^V. In this way, a function f increases along the trajectories of V with nonzero speed if, for some common $v > 0$, and for all $x \in M$, $h_x\colon t \to f(S_t^V(x))$ increases with speed at least t. The following lemma extends the deformation lemma from Morse Theory.

Lemma 1. Let M be a smooth manifold and $f\colon M \to \mathbb{R}$ be a continuous function such that M^c is compact. Suppose that M admits a nonvanishing smooth vector field V on $f^{-1}([a,b])$ such that f is increasing along the trajectories of V on the set $f^{-1}([a,b])$ with nonzero speed. Then M^b is a deformation retract of M^a.

The tautological function τ is the minimum of a compact family of smooth functions. Let P be a compact metric space (parameter space), M a compact smooth manifold with boundary and $f\colon P \times M \to \mathbb{R}$ a continuous function such that the x-derivative of f is continuous on $P \times M$. So, let $\tau = \min_{p\in P} f_p$. The set $N = \{(p,x):\ f(p,x) = \tau(x)\}$ is compact and the slices $N_x = \{p \in P:\ (p,x) \in N\}$ are upper semicontinuous, i.e., for any $x \in M$ and any open neighborhood $UN_x \supset N_x$ there exists an open neighborhood U_x of x such that, for all $x' \in U_x$, $N_{x'} \subset UN_x$. In this way, if one can perturb each x to increase τ, then this can be done globally with a minimum speed, as shown in the following lemma.

Lemma 2. Assume that, for any $x \in M$, there exists a tangent vector $V_x \in T_xM$ such that $L_{V_x} f_p > 0$. Then,

(i) for some positive v there exists a smooth vector field V on M such that $L_V f_p \geq v > 0$ in some open neighborhood of N and

(ii) along the trajectories of V, the min-function τ increases with speed at least v.

By considering the open half-spaces $H_x(p) = \{v \in T_xM : (df_p|_x, v) > 0$, over all $p \in N_x\}$, the authors define the open convex cone

$$C_x^o = \bigcap H_x(p) \subset T_xM.$$

The upper semicontinuity of N_x implies the lower semicontinuity of C_x^o for any $x \in M$ and any open set $V \subset T_xM$ with nonempty intersection with the cone. In particular, a nonempty C_x^o defines an open neighborhood of x.

Corollary 3. If the cones C_x^o are nonempty over the level set $\tau^{-1}(c)$, then c is topologically regular.

The authors call attention to the fact that a critical value is topologically regular if, for all points at the level set, the intersection of the closed half-spaces is a cone over a contractible space. It follows from Corollary 3 that, unless the level set of the tautological function $\tau^{-1}(r)$ contains a point x with $C_x^o = \varnothing$, the homotopy type of $\mathrm{Conf}(n,r)$ is locally constant at r. By Farkas' lemma, the emptiness of the cone C_x^o implies the existence of a finite collection of points $p_i \in N_x$, $i = 1,\dots,I \leq \dim(M)+1$, and positive weights $\omega_i > 0$ such that

$$(*) \qquad \sum_i \omega_i df_{p_i}|_x = 0$$

For every $\vec{x} \in \mathrm{Conf}(n)$, define the stress graph of $\vec{x}$ to be the graph embedded in $\mathbb{R}^d$ whose vertices are points $x_1, \ldots, x_n$ and boundary points $y \in \partial\mathcal{B}$ where $d(x_i, y) = r$ for some i. The edges are the pairs $\{x_i, x_j\}$ where $d(x_i, x_j) = 2r$ and $\{x_i, y\}$ where $d(x_i, y) = r$. Each edge k is assigned a weight ω_k. The points x_i are the internal points and y are the boundary points. In this manner, a stress graph is said to be balanced if it satisfies the following conditions:

(i) the mechanical stresses at each point x_i sum to zero;
(ii) the boundary mechanical stresses on each connected component sum to zero.

The configuration $\vec{x}$ is defined to be balanced if it has a balanced stress graph. An internal point is isolated if it is not in the boundary of any edges. For each point x_i, call the intersection of the stress graph with the points on the sphere $d(x_i, x) = r$ kissing points of x_i. A boundary kissing point is a kissing point on the boundary.

Lemma 4. Assume $\vec{x} \in \mathrm{Conf}(n, r)$ is balanced. Then, for the stress graph of $\vec{x}$:

(i) each nonisolated internal point is in the convex hull of its kissing points, and
(ii) each nontrivial connected component is contained in the convex hull of its boundary points.

Theorem 5. If $\vec{x} \in \mathrm{Conf}(n, r)$ is a critical point of τ with critical value r, then $\vec{x}$ is balanced and nontrivial as a point in $\mathrm{Conf}(n, r)$.

Given a sequence $L = L_1 \le \cdots \le L_d$, the authors consider the rectangular box $\mathcal{B} = \{0 \le f_m \le L_m : \ m = 1, \ldots, d\}$ (f_m stands for the orthonormal coordinate system in $\mathbb{R}^d$).

Theorem 6. For the rectangular box $\mathcal{B}$, there are no critical values of $r \in (0, L/2n)$. Therefore, $\mathrm{Conf}(n, r)$ is homotopy equivalent to $\mathrm{Conf}(n)$ in this range.

It is shown that the map $i\colon \mathrm{Conf}(n, r') \to \mathrm{Conf}(n, r)$, $r' = \frac{L}{2n} + \epsilon$, $r = \frac{L}{2n} - \epsilon$, is not a homotopy equivalence for small enough ϵ by exhibiting nontrivial classes in $H_{nd-n-d}(\mathrm{Conf}(n, r'), \mathbb{Z})$.

Let $r_* = L/2n$; in this case there are no critical values in $(r_* - \epsilon, r_* + \epsilon)$. So, $\mathrm{Conf}(n, r_* - \epsilon) \sim \mathrm{Conf}(n)$. The authors compute the Betti numbers of $\mathrm{Conf}(n, r_* + \epsilon)$. Let $N = (n-1)(d-1)$:

(i) $\beta_i(\mathrm{Conf}(n, r_* + \epsilon)) = \beta_i(\mathrm{Conf}(n, r_* - \epsilon))$, for $i \le N - 2$.
(ii) $\beta_i(\mathrm{Conf}(n, r_* - \epsilon)) = 0$, for $i \ge N$.
(iii) $\beta_{N-1}(\mathrm{Conf}(n, r_* + \epsilon)) = \beta_{N-1}(\mathrm{Conf}(n, r_* - \epsilon)) + k.n! - (n-1)!$.

Indeed, by defining $H_{n-1} = \sum_{i=1}^{n-1} 1/i$, it is shown that

$$\beta_{N-1}(\mathrm{Conf}(n, r_* + \epsilon)) = \begin{cases} (H_{n-1} + kn - 1)(n-1)!, & d = 2; \\ (kn - 1)(n-1)!, & d \ge 3. \end{cases}$$

Celso M. Doria

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR3147196 (Review) 58A12 53C21 58A10 58E40
Lu, Wen [Lu, Wen[1]] (PRC-HEF)
Morse-Bott inequalities in the presence of a compact Lie group action and applications. (English summary)
Differential Geom. Appl. **32** (2014), 68–87.

In the paper, the Morse-Bott inequalities are obtained in the presence of a compact Lie group action via Bismut and Lebeau's analytic localization techniques. Then they are used to obtain the Morse-Bott inequalities on compact manifolds with non-empty boundary by applying the generalized Morse-Bott inequalities to the doubling manifold. Indeed, the paper obtains the Morse inequalities for the multiplicities of the irreducible representations of the group. The equivariant version of the Morse Lemma is used in order to choose the geometrical data near to the singular points as simply as possible. By the equivariant Morse Lemma, for each critical submanifold B_i the following conditions are satisfied: (i) there exists a G-invariant tubular neighbourhood $N_i^- \oplus N_i^+$ of B_i and an equivariant embedding $h\colon N_i^- \oplus N_i^+ \to M$; (ii) there is an open G-invariant neighbourhood $\mathcal{B}_i$ of B_i in $N_i^- \oplus N_i^+$ such that if $Z=(Z^-,Z^+)\in\mathcal{B}_i$, then

$$f\circ h(Z^-,Z^+)=c-\frac{|\,Z^-\,|^2}{2}+\frac{|\,Z^+\,|^2}{2},$$

where c is the constant $f\,|_{B_i}$. The index n_i^- of B_i is defined as the rank of N_i^-. Let $o(N_i^-)$ denote the orientation bundle of N_i^-. By dropping the index i, consider the following spaces:

(i) $\Omega^i(B,o(N^-))$ is the space of smooth differential i-forms on B taking values in $o(N^-)$; set $\Omega(B,o(N^-))=\bigoplus_{i=0}^n\Omega^i(B,o(N^-))$.
(ii) d^B is the exterior derivative induced by the flat connection $\nabla^{o(N^-)}$.
(iii) $H^\bullet(B,o(N^-))$ is the cohomology of the Rham complex $(\Omega(B,o(N^-)),d^B)$.

The author introduces the following comparison method among the finite-dimensional G-representations: Let E_1,E_2 be two finite-dimensional G-representations and $\mathrm{Hom}_G(E_1,E_2)$ be the space of morphisms between E_1 and E_2; then $E_1\le E_2$ in the representation ring $R(G)$ if, for any irreducible representation V of G, the multiplicity of V in E_1 is smaller than the multiplicity of V in E_2; equivalently,

$$\dim(\mathrm{Hom}_G(V,E_1))\ \le\ \dim(\mathrm{Hom}_G(V,E_2)).$$

Theorem A (main theorem). Let M be a smooth m-dimensional closed and connected manifold, and let G be a compact Lie group acting smoothly on M. Assume that $f\colon M\to$

$\mathbb{R}$ is a smooth G-invariant Morse-Bott function. Then, for $k = 0, 1, \ldots, m$, we have

$$\sum_{j=0}^{k}(-1)^{k-j}H^j(M) \leq \sum_{i=1}^{r}\sum_{j=n_i^-}^{k}(-1)^{k-j}H^{j-n_i^-}(B_i, o(N_i^-))$$

in the sense defined above. When $k = m$, the equality holds:

$$\sum_{j=0}^{m}(-1)^{m-j}H^j(M) = \sum_{i=1}^{r}\sum_{j=n_i^-}^{m}(-1)^{m-j}H^{j-n_i^-}(B_i, o(N_i^-)).$$

From the main theorem, the author obtains the Morse-Bott inequalities for manifolds with non-empty boundary. Let $Y = \partial M$ and $f\colon M \to \mathbb{R}$ be a smooth function which is a Morse-Bott function in the interior of M. Denote by $H^\bullet(M, Y_r)$ the relative cohomology of M with respect to Y_r (as defined in the article).

Theorem B. The following inequalities hold for $k = 0, 1, \ldots, m$:

$$\sum_{j=0}^{k}(-1)^{k-j}\beta_j(M, Y_r) \leq \sum_{i=0}^{k}(-1)^{k-j}\mu_j,$$

where

$$\beta_j(M, Y_r) = \dim(H^j(M, Y_r)), \quad \mu_j = q_j + q_{a^+,j} + q_{r^-,j-1}.$$

The equality holds for $k = m$. *Celso M. Doria*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR3064251 (Review) 57M50 57M25
Ivanšić, Dubravko (1-MRRS-MS)
On identifying hyperbolic 3-manifolds as link complements in the 3-sphere. (English summary)
Glas. Mat. Ser. III **48(68)** (2013), *no. 1,* 173–183.

An interesting problem in 3-manifold theory is to describe sufficient and necessary conditions on a 3-manifold M that imply M is a link complement in S^3 and also establish the link. The author stresses at the beginning that a number of papers since the 1970's have shown that some hyperbolic 3-manifolds are link complements in S^3. He intends to address the question of when a particular manifold is a complement of a particular link in S^3. The author shows a method where if the starting manifold M is a non-compact hyperbolic 3-manifold, given by a side pairing polyhedron, then, if the method is successful, it produces a link in S^3 such that M is the link complement. So, there is no theorem, just a method which may or may not work. The author claims to be aware of only one case where the proof does not require knowing the link in advance—the figure eight knot complement. The method he proposes uses standard dualization of cells to convert the polyhedron and its side-pairing into a handle decomposition of the manifold. A Dehn filling is performed on the torus boundary components by adding 2-handles. By using handle moves, the handle decomposition diagram is simplified. At this point, if the diagram is an S^3 diagram then the manifold is a link complement in S^3. Tracking the longitudes and the added solid torus yields the link diagram. The author claims that the method also works for the Whitehead link, Borromean rings and some others. The method itself is highly geometrical; in order to a describe it the drawings are compulsory, so it is recommended to those interested in further details to look at the article. *Celso M. Doria*

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MR3011647 (Review) 58J99 15A66 53C55

Karapazar, Şenay (TR-ANA)

Seiberg-Witten equations on 8-dimensional manifolds with SU(4)**-structure. (English summary)**

Int. J. Geom. Methods Mod. Phys. **10** (2013), *no. 3,* 1220032, 9 *pp.*

The paper extends Witten's theorem on the existence of a solution to the Seiberg-Witten equations on a Kähler manifold N, whose real dimension is 4, over a real 8-manifold M with Spin(7)-holonomy having negative scalar curvature and the Spin^c structure associated to the canonical bundle fixed. In this particular setting, the concept of self-duality is extended by complexifying the tangent bundle and taking the decomposition $\Lambda^1 = T^*M \otimes \mathbb{C} = \Lambda^{1,0} \oplus \Lambda^{0,1}$ and also the decomposition $\Lambda^r = \bigoplus_{p+q=r} \Lambda^{p,q}$. Thus, the decomposition $\Lambda^2_{\mathbb{C}} = (\Lambda^2_7(M) \otimes \mathbb{C}) \oplus (\Lambda^2_{21}(M) \otimes \mathbb{C})$ results in the self-dual 2-forms being $\Lambda^+_2 = \Lambda^2_7 \otimes \mathbb{C}$ and the anti-self-dual being $\Lambda^-_2 = \Lambda^2_{21} \otimes \mathbb{C}$. For a fixed Spin^c-structure c on M, the complex spinor bundle S_c on M is defined and also the decomposition $S_c = S^+ \oplus S^-$, where $S^\pm$ are $\mathbb{C}L_7$-modules. Indeed, the Hermitian structure induces an SU(4) holonomy and SU(4) $\subset$ Spin(7) $\subset$ SO(8). In this way, $S_c = P_{\mathrm{Spin}^c(8)} \times_\kappa \Delta_8$, where $\kappa\colon \mathrm{Spin}^c(8) \to U(\Delta_8)$ is the spinor representation of Spin^c(8). Thus, let (M^8, J, g) be an 8-dimensional SU(4) manifold and fix a Spin^c(8) structure and a connection A in the $U(1)$-principal bundle associated with the Spin^c-structure. For $\Phi \in \Gamma(S^+)$ the author defines the Seiberg-Witten equations as the pair of equations $D_A\Phi = 0$, $F^+_A = -\frac{1}{4}\sigma(\Phi)^+$. By considering $\Phi_0 \in \Lambda^{0,0}$ the constant spinor $\Phi_0 = (0,0,0,0,0,0,0,0,1)$, the author claims that $\sigma(\Phi_0) = -4i\omega$, where ω is the Kähler form. Now, assuming that the scalar curvature R of (M^8, J, g) is negative and constant, the author proves that the spinor $\Phi_1 = \sqrt{-\frac{R}{2}}\Phi_0$ is a section of $\Lambda^{0,0}$ satisfying the Seiberg-Witten equations.

Celso M. Doria

MR2973392 (Review) 57Q10 22E20 55R10 57N13
Theriault, Stephen [Theriault, Stephen D.] (4-ABER-IM)
The homotopy types of $SU(3)$-gauge groups over simply connected 4-manifolds. (English summary)
Publ. Res. Inst. Math. Sci. **48** (2012), *no. 3,* 543–563.

Let M be a simply connected closed Riemannian 4-manifold and G be a simple, simply-connected compact Lie group. Let $\pi: P \to M$ be a principal G-bundle. The gauge group of P defined by $\mathcal{G} = \{\phi: P \to P \mid \pi \circ \phi = \phi\}$ can also be described as $\mathcal{G} = \{s: M \to G \mid s(p.g) = g^{-1}.s(p).g\}$. As $[M, BG] = \mathbb{Z}$, the principal bundles over M are classified by the second Chern class. For each $c_2(P) = k$ let $\mathcal{G}_k(M, G)$ be the gauge group of P. In [Proc. London Math. Soc. (3) **81** (2000), no. 3, 747–768; MR1781154] M. C. Crabb and W. A. Sutherland proved that there are at most finitely many distinct homotopy types of gauge groups in the family $\{\mathcal{G}_k(M, G) \mid k \in \mathbb{Z}\}$. According to [A. Kono and S. Tsukuda, J. Math. Kyoto Univ. **36** (1996), no. 1, 115–121; MR1381542], there are six homotopy types of SU(2)-gauge groups over a spin manifold, four types of SU(2)-gauge groups over a non-spin manifold, and eight homotopy types of SU(3)-gauge groups over S^4. The article under review focuses on SU(3)-gauge groups over spin and non-spin manifolds. Let $\mathcal{G}_k(M) = \mathcal{G}_k(M, \mathrm{SU}(3))$. If $a, b \in \mathbb{Z}$, let $(a, b) =$ g.c.d. of $|a|$ and $|b|$. The main theorem is the following:

Let M be a simply connected closed 4-manifold. Then the following hold:

(a) If M is a spin manifold then there is an integral homotopy equivalence $\mathcal{G}_k(M) \simeq \mathcal{G}'_k(M)$ if and only if $(24, k) = (24, k')$.

(b) If M is non-spin manifold then an integral homotopy equivalence $\mathcal{G}_k \simeq \mathcal{G}'_k$ implies that $(12, k) = (12, k')$. If $(12, k) = (12, k')$, then there is a homotopy equivalence $\mathcal{G}_k(M) \simeq \mathcal{G}'_k(M)$ after localizing rationally or at any prime.

So, there are eight distinct integral homotopy types for $\mathcal{G}_k(M)$ when M is spin and at least six integral homotopy types for $\mathcal{G}_k(M)$ when M is non-spin. The author points out that in part (b) it would be exactly six if one could use integral homotopy equivalence instead of the localized statement in part (b). Let $\mathrm{Map}_k(M, BG)$ be the component of the space of continuous unbased maps from M to BG which contains the map inducing P and let $\mathrm{Map}^*_k(M, BG)$ be the subspace of based maps $f: M \to BG$, $f(p) = *$. By defining the evaluation map $\mathrm{ev}: \mathrm{Map}_k(M, BG)$, $\mathrm{ev}(f) = f(p)$, there is the fibration $\mathrm{Map}^*_k(M, BG) \to \mathrm{Map}_k(M, BG) \xrightarrow{\mathrm{ev}} BG$. Due to the homotopy equivalence $B\mathcal{G}_k(M, G) \simeq \mathrm{Map}_k(M, BG)$, the evaluation determines a homotopy fibration sequence

$$G \xrightarrow{\partial^M_k} \mathrm{Map}^*_k(M, BG) \longrightarrow \mathcal{G}_k(M, BG) \xrightarrow{\mathrm{ev}} BG.$$

Since M is simply connected it admits a handle decomposition $M = h_0 \sqcup (\bigcup_{i=1}^d h_2^i) h_4$, where h_0 is a 0-cell, h_2^i is a 2-cell and h_4 is a 4-cell. Thus there is a homotopy cofibration sequence

$$S^3 \xrightarrow{f} \bigvee_{i=1}^{d} S^2 \longrightarrow M \xrightarrow{q} S^4 \xrightarrow{\Sigma f} \bigvee_{i=1}^{d} S^3 \qquad (1)$$

induced by the attaching maps, the pinch map q and the suspension Σf. Fixing the notation $\mathcal{G}_k = \mathcal{G}_k(S^4)$, $\Omega^3_k G = \mathrm{Map}^*_k(S^4(M, BG)) \overset{\mathrm{htpy}}{\simeq} \Omega^3_0 G$, there is a homotopy fibration diagram

$$\begin{array}{ccccccc}
 & & \prod_{i=1}^{d}\Omega^2 G & = & \prod_{i=1}^{d}\Omega^2 G & & \\
 & & \big\downarrow{\scriptstyle (\Sigma f)^*} & & \big\downarrow & & \\
G & \xrightarrow{\partial_k} & \Omega^3_k G & \longrightarrow & B\mathcal{G}_k & \longrightarrow & BG \\
\big\| & & \big\downarrow{\scriptstyle q^*} & & \big\downarrow & & \big\| \\
G & \xrightarrow{\partial_k^M} & \operatorname{Map}^*_k(M, BG) & \longrightarrow & B\mathcal{G}_k(M) & \xrightarrow{\text{ev}} & BG
\end{array}$$

In particular, ∂_k^M factors through ∂_k. Indeed, the equivalence $\partial_k \overset{\text{htpy}}{\simeq} k \circ \partial_1$ is important throughout the proof. The spin and non-spin cases are set apart because being spin is equivalent to the suspension map Σf being null homotopic (it is induced by the map f in the homotopy cofibration $S^3 \xrightarrow{f} \bigvee S^2 \longrightarrow M$). If M is not spin, then Σf is essential. Since $\pi_4(\bigvee_{i=1}^d S^3) \simeq \bigoplus_{i=1}^d \mathbb{Z}_2$, Σf is null homotopic after localizing away from 2. The following theorem from [S. D. Theriault, Algebr. Geom. Topol. **10** (2010), no. 1, 535–564; MR2602840] is a starting point:

Theorem 1. Let M be a simply connected closed 4-manifold. Then the following hold:

(i) If M is spin, then there is an integral homotopy equivalence

$$\mathcal{G}_k(M, G) \simeq \mathcal{G}_k \times \prod_{i=1}^{d} \Omega^2(G).$$

(ii) If M is not spin, then after localizing away from 2, there is a homotopy equivalence

$$\mathcal{G}_k(M, G) \simeq \mathcal{G}_k \times \prod_{i=1}^{d} \Omega^2(G).$$

Theorem 1 sets the spin case. By using [H. Hamanaka and A. Kono, Proc. Roy. Soc. Edinburgh Sect. A **136** (2006), no. 1, 149–155; MR2217512], $\mathcal{G}_k(S^4) \simeq \mathcal{G}_{k'}(S^4)$ iff $(24, k) = (24, k')$. The same theorem cannot be used to set the non-spin case since Σf may not be null homotopic. The author calls attention to the importance of determining the order of the map ∂_k in order to describe the homotopy types of $\mathcal{G}_k$. However, he stresses the difficulty of doing the same when it comes to treating the case $\mathcal{G}_k(M)$ because $\operatorname{Map}^*_k(M, BG)$ may not be an H-space and so the set of homotopy classes $[G, \operatorname{Map}^*_k(M, BG)]$ may not be a group.

Lemma 2. Suppose the map $G \xrightarrow{\partial_1} \Omega^3_0 G$ has finite order m. If $(m, k) = (m, k')$, then $\mathcal{G}_k \overset{\text{htpy}}{\simeq} \mathcal{G}_{k'}$ when localized rationally or at any prime.

Focusing on the case $G = \mathrm{SU}(3)$, the cofibration (1) induces the homotopy fibration sequence

$$\prod_{i=1}^{d} \Omega^2 \mathrm{SU}(3) \xrightarrow{(\Sigma F)^*} \Omega^3_0 \mathrm{SU}(3) \xrightarrow{q^*} \operatorname{Map}^*_k(M, B\mathrm{SU}(3)) \xrightarrow{i^*} \prod_{i=1}^{d} \Omega \mathrm{SU}(3) \xrightarrow{f^*} \Omega^2 \mathrm{SU}(3).$$

Lemma 3. Localizing at 2 the following hold:

(i) If M is spin, then $\pi_3(\operatorname{Map}^*_k(M, B\mathrm{SU}(3))) \cong \mathbb{Z}_2$ and q^* induces an isomorphism on π_3.

(ii) If M is not spin, then $\pi_3(\operatorname{Map}^*_k(M, B\mathrm{SU}(3))) \cong 0$.

The treatment of the non-spin case begins by proving the null homotopic property of the composite

$$\mathrm{SU}(3) \to \Omega^3_0 \mathrm{SU}(3) \to \Omega^3_0(\mathrm{SU}(3)) \xrightarrow{q^*} \operatorname{Map}^*_k(M, B\mathrm{SU}(3)).$$

The author calls attention to a tricky part coming from the fact that $12 \circ \partial_1$ is not null homotopic, so the composition with q^* plays a nontrivial role. Before this, he lists some properties of the map $\partial_1 \colon \mathrm{SU}(3) \to \Omega_0^3\mathrm{SU}(3)$ from [H. Hamanaka and A. Kono, op. cit.]. Let $i \colon \Sigma\mathbb{C}\mathrm{P}^2 \hookrightarrow \mathrm{SU}(3)$ be the canonical inclusion.

Lemma 4. The following hold:

(i) $\partial_1 \colon \mathrm{SU}(3) \to \Omega_0^3\mathrm{SU}(3)$ has order 24.

(ii) The composite $\Sigma\mathbb{C}\mathrm{P}^2 \xrightarrow{i} \mathrm{SU}(3) \xrightarrow{\partial_1} \Omega_0^3\mathrm{SU}(3)$ has order 24.

(iii) The composite $S^3 \to \mathrm{SU}(3) \xrightarrow{\partial_1} \Omega_0^3\mathrm{SU}(3)$ has order 6.

Combining these with some other technical results, the author proves the key proposition below asserting the null homotopy of a composite when localizing at 2.

Proposition 5. Let $t \in \mathbb{Z}$ such that $(2,t) = 1$ and let M be a simply connected non-spin 4-manifold. Then, localized at 2, the homotopic triviality of the composite

$$\mathrm{SU}(3) \xrightarrow{\partial_1} \Omega_0^3\mathrm{SU}(3) \xrightarrow{4t} \Omega_0^3\mathrm{SU}(3) \xrightarrow{q^*} \mathrm{Map}_k^*(M, B\mathrm{SU}(3))$$

is proved.

The last proposition is applied to prove that if $(12,k) = (12,k')$, then there is a homotopy equivalence $\mathcal{G}_k(M) = \mathcal{G}_{k'}(M)$ after localizing rationally or at any prime.

Under the assumption that $\mathcal{G}_k \simeq \mathcal{G}_{k'}$, the result in the main theorem is achieved by a careful analysis of the composite $\Sigma\mathbb{C}\mathrm{P}^2 \xrightarrow{i} \mathrm{SU}(3) \xrightarrow{\partial_1} \Omega_0^3\mathrm{SU}(3) \xrightarrow{q^*} \mathrm{Map}_k^*(M, B\mathrm{SU}(3))$. In the spin case, the composite $q^* \circ \partial_1 \circ i$ has order 24, and in the non-spin case the order is 12. The author basically reduces to understanding the cases when $q^* \circ \partial_1 \circ i$ has order 3 (3-primary) and $q^* \circ \partial_1 \circ i$ has order 8 (2-primary). The homotopy equivalence $\mathcal{G}_k(M) \simeq \mathcal{G}_{k'}(M)$ falls into two cases to be considered: (i) 2-primary and (ii) 3-primary. So, everything comes down to proving the following cases:

(i) 2-primary:

(a) if M is spin, then $(8,k) = (8,k')$,

(b) if M is non-spin, then $(4,k) = (4,k')$;

(ii) 3-primary:

(a) if M is spin, then $(8,k) = (8,k')$;

completing the main theorem's proof. *Celso M. Doria*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2661441 (2011g:57033) 57R57 53C27
Ishida, Masashi (J-SOPHE)
Constraints on Seiberg-Witten basic classes of anti-self-dual manifolds. (English summary)
Forum Math. **22** (2010), *no. 4,* 641–655.

Let (X,g) be a closed oriented Riemannian 4-manifold. The author addresses the question of finding necessary conditions for the existence of Seiberg-Witten basic classes on (M,g). He manages to find some numerical constraints when (X,g) is an anti-self-dual manifold with $b^+>1$. Recall that the first Chern class $c_1(\mathcal{L}_\mathfrak{s})\in H^2(X,\mathbb{Z})$ of a complex line bundle associated with a Spin^c-structure $\mathfrak{s}$ is called a Seiberg-Witten basic class of X if the Seiberg-Witten invariant for $\mathfrak{s}$ is nontrivial.

Definition 5: Let $\mathcal{H}_g^+$ be the set of all g-self-dual harmonic 2-forms on (X,g) and

$$\theta(X,g)=\min_{w\in\mathcal{H}_g^+-0}\cos^{-1}\left(\frac{\int_X|w|d\mu_g}{\mathrm{vol}_g^{1/2}(\int_X|w|^2d\mu_g)^{1/2}}\right),\quad \mathrm{vol}_g=\int_X d\mu_g,$$

$$\nu(X,g)=\min_{w\in\mathcal{H}_g^+-0}\left(\frac{\int_X|d\sqrt{w}|^2\mid d\mu_g}{\int_X|w|d\mu_g)}\right).$$

Then, let

$$\mathcal{A}(X,g)=\tfrac{1}{6}\|s_g\|_{L^2}-\nu(X,g)\cos(\theta)\sqrt{\mathrm{vol}_g}.$$

Under the notations and definitions above, the main result in the article can be stated as follows:

Theorem A (main): Let (X,g) be a closed anti-self-dual oriented manifold with

$b^+(X) > 1$. Assume $\mathfrak{s}$ is a basic class of X. Then the self-dual part c_1^+ of $c_1(\mathcal{L}_\mathfrak{s})$ satisfies

$$(c_1^+)^2 \leq \left(\frac{\mathcal{A}(X,g)}{\pi\sqrt{2}}\right)^2.$$

Next, the author uses Theorem A to prove the following vanishing results about the Seiberg-Witten invariants.

Theorem 15: Let (X,g) be a closed anti-self-dual oriented manifold with $b^+(X) > 1$. Then the Seiberg-Witten invariant vanishes for any Spin^c class $\mathfrak{s}$ if

$$2\pi^2\left(2\chi(X) + 3\tau(X) + 4d_\mathfrak{s}\right) > \mathcal{A}^2(X,g).$$

Moreover, the following holds:

(1) Suppose that $\mathfrak{c}(X) \equiv 0 \pmod 2$. If the scalar curvature s_g satisfies

$$\|s_g\|^2_{L^2} < 72\pi^2\left(2\chi(X) + 3\tau(X) + 8\right),$$

then X is simple type.

(2) Suppose that $\mathfrak{c}(X) \equiv 1 \pmod 2$. If the scalar curvature s_g satisfies

$$\|s_g\|^2_{L^2} \leq 72\pi^2\left(2\chi(X) + 3\tau(X) + 4\right),$$

then all the Seiberg-Witten invariants of X vanish. *Celso M. Doria*

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MR2440822 (2009i:28020) 28A80 28A78
Kreitmeier, Wolfgang (D-PASS-CS)

Asymptotic order of quantization for Cantor distributions in terms of Euler characteristic, Hausdorff and packing measure. (English summary)

J. Math. Anal. Appl. **342** (2008), *no. 1,* 571–584.

In this paper equality between the Euler exponent and the quantization dimension is proven under certain restrictions. Moreover, a link between the Hausdorff and the packing measure and the high-rate asymptotics of the quantization error is presented for a special sub-class of one-dimensional homogeneous Cantor sets.

{Reviewer's remarks: Definition 1.2 of the paper is the definition of the s-dimensional outer measure [see K. J. Falconer, *The geometry of fractal sets*, Cambridge Univ. Press, Cambridge, 1986; MR0867284 (p. 7)]. Its restriction to the class of all h^s-measurable sets is the s-dimensional Hausdorff measure.

{Since in the paper F is a fractal set of $\mathbb{R}^d$ but not a measurable fractal set of $\mathbb{R}^d$ it would have been better to give the exact definition.

{In the definition of the packing measure (which follows Definition 1.3) the sets E_i are not required to be measurable sets, so it is not clear why is $P^s(F)$ a countably additive probability.}

Serena Doria

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2192203 (2006k:57085) 57R57 57M60 57R15 57S17
Liu, Ximin (J-TOKYOGM)

On S_3-actions on spin 4-manifolds. (English summary)

Carpathian J. Math. **21** (2005), *no. 1-2,* 137–142.

Let X be a smooth, closed, connected Spin 4-manifold. One of the main conjectures concerning Spin 4-manifolds claims that

$$b_2(X) \geq \frac{11}{8}|\sigma(X)|.$$

The formula justifies its name, the $\frac{11}{8}$-conjecture. Since the intersection form Q_X of X is equivalent to

$$-2kE_8 \oplus mH, \quad k, m \geq 0,\ H = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

the $\frac{11}{8}$-conjecture is equivalent to the claim that $m \geq 3k$. Donaldson has proved that if $k > 0$, then $m \geq 3$. Since $K3$ satisfies $m = 3$ and $k = 1$ the $\frac{11}{8}$-conjecture is optimal. Later Furuta proved that whenever $b_1(X) = 0$, then $m \geq 2k + 1$.

Using Furuta's techniques, some other authors obtained similar results for G-manifolds, where G is a group acting on X preserving the Spin structure. Considering the case where X is an S_3-manifold (S_3 is the symmetric group) and the action is even, the article's main theorem is:

Theorem. Let X be a smooth Spin 4-manifold with $b_1(X) = 0$ and non-positive signature. Let $k = -\sigma(X)/16$ and $m = b_2^+$. If S_3 acts on X so that the action is of Spin even type, $b_2^+(X/\langle x_2\rangle)>0$, $b_2^+(X/\langle x_1\rangle)>0$, and $b_2^+(X)\neq b_2^+(X/\langle x_1\rangle)>0$, then

$$2k+2\leq m.$$

(Here x_i are generators of S_3.)

As in [M. Furuta, Math. Res. Lett. **8** (2001), no. 3, 279–291; MR1839478], the author explores the fact that the monopole map is $\mathrm{Pin}_2\times S_3$-equivariant. The monopole map is the map $\mathcal{D}+\mathcal{Q}\colon V\to W$,

$$\mathcal{D}(a,\phi)=(D\phi,\rho(d^+a),d^*a),$$
$$\mathcal{Q}(a,\phi)=(\rho(a)\phi,\phi\otimes\phi-\tfrac{1}{2}|\phi|^2I,0),$$

where $V=\Gamma(i\Omega^1(X)\oplus S^+)$ and $W=\Gamma(S^-\oplus i\mathrm{su}(S^+)\oplus\Omega^0(X))$. Thus, the author computes the index of the twisted Dirac operator D and the character formula for the K-theoretic degree. This is performed in the following setting: let V and W be G-representations for some compact Lie group G. Let BV and BW denote the balls in V and W, respectively, and $f\colon BV\to BW$ a G-map preserving the boundaries $SV=\partial(BV)$ and $SW=\partial(BW)$. By definition $K_G(V)=K_G(BV,SV)$; the Thom isomorphism theorem claims that $K_G(V)$ is a free $R(G)$-module generated by the Bott class $\lambda(V)$. Applying the K-theory functor to f we get a map $f^*\colon K_G(W)\to K_G(V)$. Thus there exists $\alpha_f\in R(G)$, known as the K-theoretic degree of f, such that $f^*(\lambda(W))=\alpha_f.\lambda(V)$.

Now let V_g and W_g denote subspaces invariant by an element $g\in G$, and $V_g^\perp$ and $W_g^\perp$ their orthogonal complement, respectively. Let $f^g\colon V_g\to W_g$ be the restriction $f_g=f|_{V_g}$ and let $d(f^g)$ denote the ordinary topological degree of f^g. For any $\beta\in R(G)$, let $\lambda_{-1}\beta=\sum_i(-1)^i\lambda^i\beta$ be the alternating sum of exterior powers. By a theorem of T. tom Dieck, the character formula for the degree α_f, where $f\colon BV\to BW$ is a G-map preserving boundaries, is given by

$$\mathrm{tr}_g(\alpha_f)=d(f^g)\mathrm{tr}_g(\lambda_{-1}(W_g^\perp-V_g^\perp)).$$

Thus, in the scenario described above, the monopole map acts as a $\mathrm{Pin}_2\times S_3$-map.

It follows from the main theorem that a homotopy $K3$ manifold cannot admit a nontrivial Spin S_3 action of even type. *Celso M. Doria*

MR2069840 (2005j:70063) 70S15 53C80 58E30 70G45
Sandovici, Adrian (NL-GRON-DMC)

A rheonomic gauge theory. (English summary)

Carpathian J. Math. **19** (2003), *no. 2,* 111–134.

Let M be a smooth n-dimensional manifold and consider the vector bundle $(E=\mathbb{R}\times M,\pi,\mathbb{R}\times M)$ over $\mathbb{R}\times M$, with local coordinates (t,x,y) on E. A rheonomic gauge transformation of the bundle E is a pair (f_1,f_2) of diffeomorphisms $f_1\colon E\to E$, $f_2\colon M\to M$ such that $\pi\circ f_1=f_2\circ\pi$. The author introduces the concept of a nonlinear connection in the classical fashion by specifying the way it transforms when a transformation of coordinates is applied; he also introduces the concept of a semispray as being a vector field S on E that in every chart takes the form

$$S = \frac{\partial}{\partial t} + y^i \frac{\partial}{\partial x^i} - 2G^i(t,x,y)\frac{\partial}{\partial y^i},$$

where $G^i(t,x,y)$ are smooth real functions. Nonlinear connections and semisprays are related in the following way:

(1) If N is a nonlinear connection on E given by the local coefficients (N^i_0, N^j_1), then

$$S_0 = \frac{\partial}{\partial t} + y^i \frac{\partial}{\partial x^i} - N^i_k \frac{\partial}{\partial y^i}$$

and

$$\frac{\partial}{\partial t} + y^i \frac{\partial}{\partial x^i} - (N^i_0 + N^i_k y^k)\frac{\partial}{\partial y^i}$$

are semisprays on E.

(2) If S is a semispray on E defined locally by $(G^i(t,x,y))$, then the pair of functions $(\lambda \frac{\partial G^i}{\partial t}, \frac{\partial G^i}{\partial y^j})$, $\lambda \in \mathbb{R}$, defines a class of nonlinear connections on E.

Next, the author introduces the class of gauge nonlinear connections determined by a gauge time-dependent Lagrangian on E. A time-dependent Lagrangian $L\colon E \to \mathbb{R}$ is said to be regular if the matrix $(g_{ij}) = \left(\frac{1}{2}\frac{\partial^2 L}{\partial y^i \partial y^j}\right)$ is of rank n on E. Then a rheonomic Lagrange space is defined as a pair $(M, L(t,x,y))$, where L is a regular time-dependent Lagrangian such that the quadratic form with coefficients $g_{ij}(t,x,y)$ has constant signature.

In order to derive the equations of motion for a rheonomic Lagrangian on E, it is assumed that there exist p smooth physical fields $Q^A(t,x,y)$ on E and

$$G = dt \otimes dt + g_{ij} dx^i \otimes dx^j + g_{ij} dy^i \otimes dy^j$$

is a gauge metric structure on E induced by $g_{ij}(t,x,y)$, where g_{ij} is the fundamental tensor field of the rheonomic Lagrange space (M, L). It is also assumed that on E there exists a gauge nonlinear connection. Thus, the equations of motion are obtained for a Lagrangian $L_0(t,x,y) = L(Q^A, \frac{\partial Q^A}{\partial x^\alpha}, \frac{\partial Q^A}{\partial y^i})$. Finally, conservation laws are derived by exploiting the G-invariance of L_0, where G is a Lie group. *Celso M. Doria*

MR2044546 (2005e:57050) 57M50 57M25 57N10

Otal, Jean-Pierre (F-ENSLY-PM)

Les géodésiques fermées d'une variété hyperbolique en tant que nœuds. (French. English, French summaries) [Closed geodesics in a hyperbolic manifold, viewed as knots]

Kleinian groups and hyperbolic 3-manifolds (Warwick, 2001), 95–104, *London Math. Soc. Lecture Note Ser.*, 299, *Cambridge Univ. Press*, *Cambridge*, 2003.

The paper addresses the question of the unknottedness and unlinkedness of closed curves in a hyperbolic 3-manifold. The main statement claims that whenever M is a hyperbolic 3-manifold homotopically equivalent to a surface of genus g, then there exists a constant

$c(g) > 0$ such that any collection of closed, primitive geodesics of M whose lengths are shorter than $c(g)$ is unlinked in M. The concepts of unknotted and unlinked curves in a 3-manifold M homotopically equivalent to a surface of genus g is defined as follows.

Definition: Let S be a surface (not necessarily compact) and $f\colon S \to M$ an embedding in M. A closed non-self-intersecting curve $\gamma \subset M$ is unknotted in M, with respect to f, if f is properly isotopic to an embedding f' such that $\gamma \subset f'(S)$.

First of all, the author proves that whenever the length of a closed geodesic γ^* is shorter than the constant $c(g)$, then γ^* is unknotted in M. For the case of a single curve, the author reduces the main statements to the same claim restricted to a neighbourhood N of $f(S)$ in M. So, the proof is carried out by applying a construction known as Papakyriakopoulos's Tower.

In order to extend the result for a collection of closed, primitive (generator of $\pi_1(M)$) geodesics, the author introduces the following concept of unlinking.

Definition: Let $L \subset M$ be a locally finite set of mutually disjoint embedded curves. L is unlinked in M if there exists a homeomorphism between M and $S \times (-\infty, \infty)$ such that each component of L is contained in one of the surfaces $S \times \{i\}$, $i \in \mathbb{Z}$.

The general position argument is carried out to prove the main statement for a collection of closed primitive geodesics.

As an application, the author considers the case where M has the homotopy type of a compact surface and N is its Nielsen core.

Theorem: Let γ^* be a closed primitive geodesic of M which is homotopically equivalent to a simple closed curve in ∂N and whose length is shorter than a constant $\epsilon(3)$. Then γ^* is unknotted with respect to the embedding $\partial N \to M$.

{For the collection containing this paper see MR2044542}

Celso M. Doria

MR1987794 (2004k:57042) 57R58 57R57 58J60
Carey, Alan L. (5-ANU-MS); **Wang, Bai Ling** (5-ADLD)
Seiberg-Witten-Floer homology and gluing formulae. (English summary)
Acta Math. Sin. (Engl. Ser.) **19** (2003), *no. 2,* 245–296.

The paper gives a very technical and clear construction of the Seiberg-Witten-Floer homology for a closed oriented 3-manifold endowed with a non-torsion Spin^c structure. Moreover, it relates the homology with the Seiberg-Witten invariants. The same structure present in Floer homology [S. K. Donaldson, *Floer homology groups in Yang-Mills theory*, Cambridge Univ. Press, Cambridge, 2002; MR1883043] is explored, namely, a sort of Morse-Smale-Witten complex [D. A. Salamon, Bull. London Math. Soc. **22** (1990), no. 2, 113–140; MR1045282] is constructed. In categorical terms, it is proved that the Seiberg-Witten theory is a topological quantum field theory [F. Quinn, in *Geometry and quantum field theory (Park City, UT, 1991)*, 323–453, Amer. Math. Soc., Providence, RI, 1995; MR1338394].

Let X be a closed 4-manifold admitting a decomposition $X = X_+ \cup_Y X_-$ along a 3-manifold Y ($X_\pm$ are 4-manifolds with boundary $\partial X_\pm = Y$). The main theorem is concerned with the relationship between the SW-invariant of X and the SW-invariants of each component $X_\pm$. In order to obtain such a relationship, it is shown that each SW-invariant of $X_\pm$ takes its value in the $\mathbb{Z}$-module $HF^{\mathrm{SW}}_{*,[\mathrm{Im}(i^*_+)]}(Y,\mathfrak{t})$ associated to the 3-manifold Y. ($i_\pm$ are the boundary inclusion maps.) By applying the natural pairing, it is shown that the SW-invariant of X can be obtained from the SW-invariants of X_+ and of X_-.

The pairing constructions are performed as follows in the steps below.

A Spin^c structure on Y is given by a pair $\mathfrak{t}=(W,\rho)$, where $W=P_{\mathrm{SO}_3}\times_\rho V$ is the associated bundle to the tangent bundle of Y induced by the irreducible Clifford representation $\rho\colon \mathrm{Cl}_3\to \mathrm{End}(V)$. Let $\mathcal{A}$ be the space of L^2_1-connections on the determinant bundle of $\mathfrak{t}$ and $\Gamma(W)$ the space of L^2_1-sections of W; the L^2_1-configuration space for the Seiberg-Witten equations on Y is $\mathcal{C}=\mathcal{A}\times\Gamma(W)$.

1. The Seiberg-Witten-Floer complex defined on Y.

Fixing a C^∞ connection A_0 on $\det(\mathfrak{t})$ and a co-closed 2-form $\eta\in\Omega^2(Y,i\mathbb{R})$, which is L^2_2-integrable, $*\eta$ trivial in $H^1(Y,i\mathbb{R})$, the Chern-Simons-Dirac $\mathcal{C}_\eta\colon\mathcal{C}\to\mathbb{R}$ functional on the configuration space

$$\mathcal{C}_\eta(A,\psi)= -\frac{1}{2}\int_Y (A-A_0)\wedge(F_A-F_{A_0}-*2\eta)+\int_Y\langle\psi,\partial_A\psi\rangle d\mathrm{vol}_Y$$

is considered.

It is shown that for a generic η (in the sense of the Sard-Smale theorem), $\mathcal{C}_\eta$ is a Morse-Smale function, which allows one to define a relative Morse index $\mu(p,q)$ between critical points $p,q\in\mathcal{C}$. Since the relative indices are finite, by fixing a critical point $p\in\mathcal{C}$ for each $k\in\mathbb{N}$, the $\mathbb{Z}$-modules $H^{\mathrm{SW}}_k(Y,\mathfrak{t})$ and the Morse-Witten-Floer complex $(H^{\mathrm{SW}}_*(Y,\mathfrak{t}),\partial)$ are considered as follows:

$$H^{\mathrm{SW}}_k(Y,\mathfrak{t})=\left\{\sum_{i=1}^{m\le\infty} l_i\langle p_i\rangle|\mu(p,p_i)=k,\ l_i\in\mathbb{Z}\right\};$$

$$H^{\mathrm{SW}}_*(Y,\mathfrak{t})=\bigoplus_{i=0} H^{\mathrm{SW}}_k(Y,\mathfrak{t}).$$

The boundary operator ∂ is the canonical one defined in Morse-Smale-Witten (Floer) theory [D. A. Salamon, op. cit.; S. K. Donaldson, op. cit.]. Thus, it is shown that:

(a) $HF^{\mathrm{SW}}_*(Y,\mathfrak{t})$ is a topological invariant of $(Y,\mathfrak{t})$ and is a $Z_{d(\mathfrak{t})}$-graded abelian group, where $d(\mathfrak{t})=\mathrm{g.c.d.}\{c_1(\mathfrak{t})(\sigma)|\sigma\in H_2(Y,\mathbb{Z})\}$.

(b) There is an action of $\mathbb{A}(Y)=\mathrm{Sym}^*(H_0(Y,\mathbb{Z}))\otimes\Lambda^*(H_1(Y,\mathbb{Z})/\mathrm{torsion})$ on $HF^{\mathrm{SW}}_*(Y,\mathfrak{t})$ with elements in $H_0(Y,\mathbb{Z})$ and $H_1(Y,\mathbb{Z})/\mathrm{torsion}$ decreasing degree in $HF^{\mathrm{SW}}_*(Y,\mathfrak{t})$ by 2 and 1, respectively.

(c) For $(-Y,-\mathfrak{t})$, where $-Y$ is Y with reversed orientation and $-\mathfrak{t}$ is the induced Spin^c structure, the corresponding Seiberg-Witten-Floer complex $C_*(-Y,-\mathfrak{t})$ is the dual complex of $C_*(Y,\mathfrak{t})$. There is a natural pairing

$$\langle\cdot,\cdot\rangle\colon HF^{\mathrm{SW}}_*(Y,\mathfrak{t})\times HF^{\mathrm{SW}}_{-*}(-Y,-\mathfrak{t})\to\mathbb{Z} \tag{1}$$

such that $\langle z\cdot\Xi_1,\Xi_2\rangle=\langle\Xi_1,z\cdot\Xi_2\rangle$ for any $z\in\mathbb{A}(Y)\simeq\mathbb{A}(-Y)$ and any cycles $\Xi_1\in HF^{\mathrm{SW}}_*(Y,\mathfrak{t})$ and $\Xi_2\in HF^{\mathrm{SW}}_*(Y,\mathfrak{t})$, respectively.

(d) For any subgroup $K\subseteq\mathrm{Ker}(c_1(\mathfrak{t}))\subset H^1(Y,\mathbb{Z})$, there is a variant of Seiberg-Witten-Floer homology denoted by $HF^{\mathrm{SW}}_{*,[K]}(Y,\mathfrak{t})$. $HF^{\mathrm{SW}}_{*,[K]}(Y,\mathfrak{t})$ is a topological invariant and a $\mathbb{Z}$-graded $\mathbb{A}$-module. In addition, the pairing extends to these groups and

$$HF^{\mathrm{SW}}_{m,[\mathrm{Ker}(c_1(\mathfrak{t}))]}(Y,\mathfrak{t})\simeq HF^{\mathrm{SW}}_{m\ (\mathrm{mod}\ (d(\mathfrak{t})))}(Y,\mathfrak{t}),\quad\forall m\in\mathbb{Z}.$$

2. Computing the Seiberg-Witten invariants of manifolds with tubular ends.

Let $(X_+, \mathfrak{s}_+)$ be a 4-manifold with a cylindrical end modelled on $(Y, \mathfrak{t})$, which means that over the end $[2, \infty) \times Y$ there is a fixed isomorphism between the restriction of $\mathfrak{s}_+$ and the pull-back Spin^c structure of $\mathfrak{t}$. In addition, it is assumed that $c_1(\det(\mathfrak{t}))$ is non-torsion. Let $i: Y \hookrightarrow X$ be an embedding. Then the Seiberg-Witten series is a linear functional

$$\mathrm{SW}_{X_+}(\mathfrak{s}_+, \cdot): \mathbb{A}(X_+) \to HF^{\mathrm{SW}}_{*,[\mathrm{Im}(i_+^*)]}(Y, \mathfrak{t}),$$

where $\mathrm{Im}(i_+^*)$ is the range of the homomorphism $i_+^*: H^1(X_+, \mathbb{Z}) \to H^1(Y, \mathbb{Z})$.

Therefore, in this case the Seiberg-Witten invariants take values in the homology groups $HF^{\mathrm{SW}}_{*,[\mathrm{Im}(i_+^*)]}(Y, \mathfrak{t})$.

3. Computing the Seiberg-Witten invariant of $X = X_+ \cup X_-$.

Let $(X, \mathfrak{s})$ be a 4-manifold with Spin^c structure $\mathfrak{s}$ and a smooth separating 3-manifold Y such that $[-2, 2] \times Y$ is embedded in X and $\mathfrak{t} = \mathfrak{s}|_Y$ is a non-torsion class. Consider a 1-parameter family of metrics $\{g_R\}_{R>0}$ on X such that for each $X(R) = (X, g_R)$, there are an isometrically embedded submanifold $([-R-2, R+2] \times Y, dt^2 + g_Y)$ and two 4-manifolds $X_\pm$ obtained by setting $X(R) = X_+(R) \cup_Y X_-(R)$. As $R \to \infty$, $X(R)$ has a geometric limit of two 4-manifolds with cylindrical ends, denoted by $X_\pm(\infty)$, endowed with Spin^c structures $\mathfrak{s}_\pm$ induced from $\mathfrak{s}$. Let $i_\pm$ be the boundary embedding maps of Y in $X_\pm(0)$, and $\mathrm{Im}(i_\pm^*)$ the ranges of the maps $H^1(X_\pm(0, \mathbb{Z})) \to H^1(Y, \mathbb{Z})$.

With these notations understood, the relative invariants for $(X_\pm, \mathfrak{s}_\pm)$ are linear functionals

$$\mathrm{SW}_{X_\pm}(\mathfrak{s}_\pm, \cdot): \mathbb{A}(X_\pm) \to HF^{\mathrm{SW}}_{*,[\mathrm{Im}(i_\pm^*)]}(\pm Y, \pm\mathfrak{t}).$$

The set of Spin^c structures on $X(R)$, obtained by gluing $\mathfrak{s}_\pm$ along $(Y, \mathfrak{t})$, is represented by

$$\mathrm{Spin}^c(X, \mathfrak{s}_\pm) = \left\{ s_+ \#_{[u]} s_- \mid [u] \in \frac{H^1(Y, \mathbb{Z})}{\mathrm{Im}(i_+^*) + \mathrm{Im}(i_-^*)} \right\}.$$

Assuming that $b_1(Y) > 0$ and $c_1(\mathfrak{t}) \neq 0$, the main theorem results from analytical gluing techniques performed along $(Y, \mathfrak{t})$.

Let $X = X_+ \cup_Y X_-$ be a 4-manifold, where $\partial X_\pm = Y$ and $X_\pm$ are endowed with Spin^c structures $\mathfrak{s}_\pm$ which restrict to $\mathfrak{t}$ on Y. Then the Seiberg-Witten invariants for $(X, \mathfrak{s}_+ \#_{[u]} \mathfrak{s}_-)$ can be expressed as

$$\mathrm{SW}_X(\mathfrak{s}_+ \#_{[u]} \mathfrak{s}_-, z_+ z_-) = \langle [u](\pi_+(\mathrm{SW}_{X_+}(\mathfrak{s}_+, z_+)), \pi_-(\mathrm{SW}_{X_-}(\mathfrak{s}_-, z_-))) \rangle,$$

where

(a) $[u]$ acts on $HF^{\mathrm{SW}}_{*,[\mathrm{Im}(i_+^*)+\mathrm{Im}(i_-^*)]}(Y, \mathfrak{t})$, $z_\pm \in \mathbb{A}(X_\pm)$,

(b) $\pi_\pm$ is the homomorphism $\pi_\pm: HF^{\mathrm{SW}}_{*,[\mathrm{Im}(i_\pm^*)]}(\pm Y, \pm\mathfrak{t}) \to HF^{\mathrm{SW}}_{*,[\mathrm{Im}(i_\pm^*)+\mathrm{Im}(i_-^*)]}(\pm Y, \pm\mathfrak{t})$,

(c) the pairing on the right-hand side is the natural pairing, defined on

$$HF^{\mathrm{SW}}_{*,[\mathrm{Im}(i_+^* + \mathrm{Im}(i_-^*))]}(Y, \mathfrak{t}) \times HF^{\mathrm{SW}}_{*,[\mathrm{Im}(i_+^*)+\mathrm{Im}(i_-^*)]}(-Y, -\mathfrak{t}),$$

with degrees in $HF^{\mathrm{SW}}_{*,[\mathrm{Im}(i_+^*)+\mathrm{Im}(i_-^*)]}(-Y, -\mathfrak{t})$ shifted by

$$d_X(\mathfrak{s}) = \frac{1}{4}\left(c_1(\mathfrak{s})^2 - (2\chi(X) + 3\sigma(X))\right) = \deg(z_+) + \deg(z_-).$$

The main theorem is extended to the case $b_2^+(X) = 1$ by assuming some extra hypotheses about the orientation of the moduli spaces. *Celso M. Doria*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1900887 (2003b:57037) 57R17 53D35
Cho, Yong Seung (KR-EWHA); **Joe, Dosang** (KR-POST)
Anti-symplectic involutions with Lagrangian fixed loci and their quotients. (English summary)
Proc. Amer. Math. Soc. **130** (2002), *no. 9,* 2797–2801 (*electronic*).

Let X be a closed symplectic 4-manifold endowed with an anti-symplectic involution ($\tau\colon X \to X$ and $\tau^*\omega = -\omega$). Let X^τ be the fixed locus of τ.

The main result obtained by the authors is the following: Assume that $b_2^+(X) \geq 2$ and that the fixed locus X^τ is a disjoint union of Lagrangian surfaces. If one of the components of X^τ has genus $g \geq 2$, then X/τ has vanishing Seiberg-Witten invariants. The lines of the proof are as follows:

Let Σ_g be the Lagrangian component of X^τ. By Weinstein's Lagrangian neighbourhood theorem [D. McDuff and D. A. Salamon, *Introduction to symplectic topology*, Oxford Univ. Press, New York, 1995; MR1373431], the normal bundle of Σ_g is isomorphic to the cotangent bundle of σ_g, and so $[\Sigma_g]\cdot[\Sigma_g] = 2g-2$. The quotient surface $\tilde{\Sigma_g} = \Sigma_g/\tau$ has self-intersection $[\tilde{\Sigma_g}]\cdot[\tilde{\Sigma_g}] = 4g-4$. So $g \geq 2 \Longrightarrow [\tilde{\Sigma_g}]^2 > 0$; this violates the adjunction inequality [P. S. Ozsváth and Z. Szabó, Ann. of Math. (2) **151** (2000), no. 1, 93–124; MR1745017; P. B. Kronheimer and T. S. Mrowka, Math. Res. Lett. **1** (1994), no. 6, 797–808; MR1306022]. Therefore, the SW-invariants must vanish.

Since the hypothesis of existence of such an anti-symplectic involution is rather nontrivial, the authors show some examples. They construct a hypersurface X_d, of degree $d \geq 4$ ($d < 4 \Longrightarrow b_2^+(X_d) = 1$), in $\mathbb{C}\mathrm{P}^3$, defined by a homogeneous polynomial $F(x,y,z) \in \mathbb{R}[x,y,z]$. In this way, X_d admits an anti-holomorphic involution whose locus is the set of real solutions $X^\tau = X \cap \mathbb{R}\mathrm{P}^3$. They consider two cases: If d is even, then $X^\tau = X \cap \mathbb{R}\mathrm{P}^3 \simeq \Sigma_g$. If d is odd, then $X^\tau = X \cap \mathbb{R}\mathrm{P}^3 \simeq \Sigma_g \cup \mathbb{R}\mathrm{P}^2$.

Another example considered by the authors is the 4-manifold $X = (\Sigma_g \times \Sigma_g, \omega \oplus \omega)$. Let $f\colon \Sigma_g \to \Sigma_g$, $f^*\omega = -\omega$, and define $\tau_f\colon X \to X$ as $\tau_f(x,y) = (f^{-1}(y), f(x))$; then τ_f is an anti-symplectic involution and $X^{\tau_f} = \{(x,y) \mid y = f(x)\} \simeq \Sigma_g$. *Celso M. Doria*

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Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.